Kvadratická s parametrom

$x^2+6x+t=0$ $\quad a=1 \quad b=6 \quad c=t$

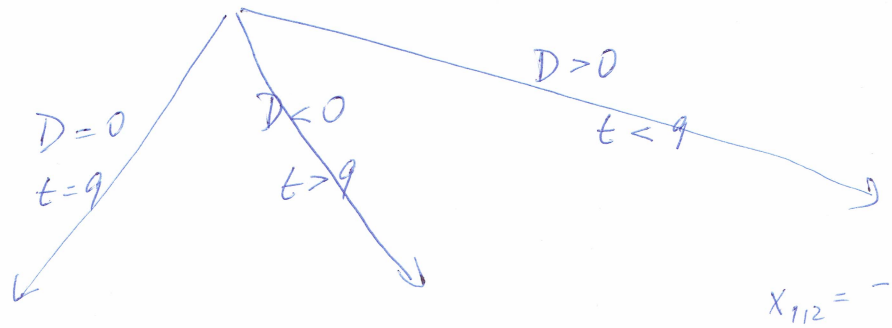
$D=6^2-4.1.t=36-4t=4.(9-t)$

$D=0 \Leftrightarrow t=9$

$D<0 \Leftrightarrow 9-t<0 \Leftrightarrow t>9$

$D>0 \Leftrightarrow 9-t>0 \Leftrightarrow t<9$

23/199a



$x=-\frac{b}{2a}=\frac{-6}{2.1}=\underline{-3}$ $\quad \underline{x\in\emptyset}$

$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}=\frac{-6\pm\sqrt{36-4t}}{2.1}=$

$=\frac{-6\pm 2\cdot\sqrt{9-t}}{2}=\underline{-3\pm\sqrt{9-t}}$

t	x
$(-\infty;9)$	$-3\pm\sqrt{9-t}$
9	-3
$(9;\infty)$	$\emptyset$

Kvadratická s parametrom

$2x^2 + t = 0$ $\quad a = 2 \quad b = 0 \quad c = t$

$D = b^2 - 4ac = 0^2 - 4 \cdot 2 \cdot t = -8t$

ak $D = 0$, $+j. \; -8t = 0$, $+j. \; \boxed{t = 0}$

1 koreň:

$x = \frac{-b}{2a} = \frac{-0}{2 \cdot 2} = \boxed{0}$ ①

ak $D < 0$, $+j. \; -8t < 0$, $\boxed{t > 0}$

~~0~~ koreňov ②

ak $D > 0$, $+j. \; \boxed{t < 0}$

$x_{1,2} = \frac{-b \pm \sqrt{D}}{2}$

$x_{1,2} = \frac{-0 \pm \sqrt{-8t}}{2} =$

$= \frac{\pm 2 \cdot \sqrt{-2t}}{2} =$

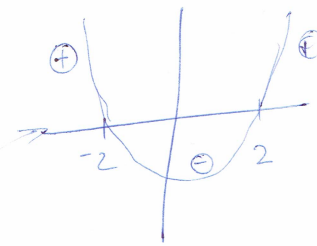
$= \boxed{\pm \sqrt{-2t}}$ ③

t	x	
0	0	①
$(0; \infty)$	$\emptyset$	②
$(-\infty; 0)$	$\pm\sqrt{-2t} = \{-\sqrt{-2t}; \sqrt{-2t}\}$	③

23/1a

$x^2 + sx + 1 = 0$ $\quad a = 1 \quad b = s \quad c = 1$

$D = s^2 - 4 \cdot 1 \cdot 1 = s^2 - 4 = (s-2)(s+2)$



$D = 0$:

$(s-2) \cdot (s+2) = 0$

$s = 2$ alebo $s = -2$

$x = -\frac{b}{2a} = -\frac{s}{2 \cdot 1} = -\frac{s}{2}$

$D < 0$

$(s-2)(s+2) < 0$

$-2 < s < 2$

$x \in \emptyset$

$D > 0$

$s < -2$ alebo $s > 2$

$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-s \pm \sqrt{s^2 - 4}}{2 \cdot 1}$

s	x
2	$-\frac{s}{2} = -1$
-2	$-\frac{s}{2} = 1$
$(-2; 2)$	$\emptyset$
$(-\infty; -2) \cup (2; \infty)$	$\frac{-s \pm \sqrt{s^2 - 4}}{2}$

23/1a

KVADRATICKÁ S PARAMETROM

$9x^2 + px - 6 = 2x - p$

$9x^2 + px - 2x + p - 6 = 0$

$9x^2 + (p-2)x + (p-6) = 0 \qquad a = 9 \quad b = p+2 \quad c = p-6$

Rozbor $D = 0$, $D \lessgtr 0$

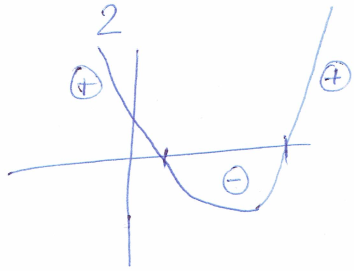
$D = (p+2)^2 - 4 \cdot 9 \cdot (p-6) = p^2 - 4p + 4 - 36p + 216 = p^2 - 40p + 220$

$D = 0 \iff p^2 - 40p + 220 = 0$

$D_p = (-40)^2 - 4 \cdot 220 = 1600 - 880 = 720$

$p_{1,2} = \dfrac{-(-40) \pm \sqrt{720}}{2 \cdot 1} = \dfrac{+40 \pm 4 \cdot 3 \cdot \sqrt{5}}{2} =$

$= 20 \pm 6 \cdot \sqrt{5}$



$D < 0 \iff p \in (20 - 6\sqrt{5}; 20 + 6\sqrt{5})$

$D > 0 \iff p \in (-\infty; 20 - 6 \cdot \sqrt{5}) \cup (20 + 6\sqrt{5}; \infty)$

$D = 0$

$p = 20 \pm 6\sqrt{5}$

$x = \dfrac{-b}{2a} = \dfrac{-p+2}{2 \cdot 9}$

$x = \dfrac{-p+2}{18}$

$D < 0$

$p \in (20 - 6\sqrt{5}; 20 + 6\sqrt{5})$

$x \in \emptyset$

$D > 0$

$p \in (-\infty; 20 - 6\sqrt{5}) \cup (20 + 6\sqrt{5}; \infty)$

$x_{1,2} = \dfrac{-(p+2) \pm \sqrt{p^2 - 40p + 220}}{2 \cdot 9}$

p	x
$(-\infty; 20 - 6\sqrt{5}) \cup (20 + 6\sqrt{5}; \infty)$	$\dfrac{-p - 2 \pm \sqrt{p^2 - 40p + 220}}{18}$
$(20 - 6\sqrt{5}; 20 + 6\sqrt{5})$	$\emptyset$
$20 + 6 \cdot \sqrt{5}$	$\dfrac{-p+2}{18} = \dfrac{-20 - 6\sqrt{5} + 2}{18} = -1 - 3\sqrt{5}$
$20 - 6\sqrt{5}$	$\dfrac{-p+2}{18} = \dfrac{-20 + 6\sqrt{5} + 2}{18} = -1 + 3\sqrt{5}$

Kvadratická s parametrom

$x^2 + 4\lambda x + 36 = 0$

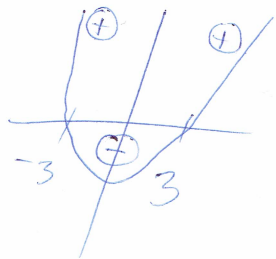
$a = 1 \quad b = 4\lambda \quad c = 36$

$D = (4\lambda)^2 - 4 \cdot 1 \cdot 36 = 16\lambda^2 - 144 = 16 \cdot (\lambda^2 - 9) = 16 \cdot (\lambda + 3) \cdot (\lambda - 3)$

$D = 0 \iff 16 \cdot (\lambda + 3)(\lambda - 3) = 0 \iff \lambda = \pm 3$

$D < 0 \iff \lambda \in (-3; 3)$

$D > 0 \iff \lambda \in (-\infty; -3) \cup (3; \infty)$



$D = 0$ ... $\lambda = \pm 3$

$x_q = -\frac{b}{2a} = \frac{-4\lambda}{2} = \underline{\underline{-2\lambda}}$

$D < 0$ ... $\lambda \in (-3; 3)$

$\underline{\underline{\emptyset}}$

$D > 0$ ... $\lambda \in (-\infty; -3) \cup (3; \infty)$

$x_{1,2} = \frac{-4\lambda \pm 4\sqrt{\lambda^2 - 9}}{2}$

$\underline{\underline{x_{1,2} = -2\lambda \pm 2\sqrt{\lambda^2 - 9}}}$

λ	x
3	$-2\lambda = -6$
-3	$-2\lambda = 6$
$(-3; 3)$	$\emptyset$
$(-\infty; -3) \cup (3; \infty)$	$-2\lambda \pm 2\sqrt{\lambda^2 - 9}$

Kvadratická s parametrom

$x^2 - (m+3).x + m - 13 = 0$

$a = 1 \qquad b = -(m+3) \qquad c = m - 13$

$D = [-(m+3)]^2 - 4.1.(m-13) = m^2 + 6m + 9 - 4m + 52 =$

$= m^2 + 2m + 61$

$D = 0 \Leftrightarrow m^2 + 2m + 61 = 0$

$D_m = 2^2 - 4.61 < 0$

$\Downarrow$

pre ľubovoľné $m \in R$ nie je rovné 0

teda $D \neq 0$ pre žiadne $m \in R$

$m^2 + 2m + 61 = (m^2 + 2m + 1) + 60 = (m+1)^2 + 60$

t.j. lebo diskriminant je kladný pre $\forall m \in R$

$D > 0$ pre $\forall m \in R$

$$x_{1,2} = \frac{+(m+3) \pm \sqrt{m^2 + 2m + 61}}{2.1}$$

m	x
R	$\frac{m+3 \pm \sqrt{m^2+2m+61}}{2}$

KVADRATICKÁ S PARAMETROM

$p^2x^2 - 2x + p = 0$

↳ ak $p = 0 \Rightarrow$ (lin.) $-2x = 0 \Rightarrow x = 0$

↳ ak $p \neq 0 \Rightarrow$ (kvadr.) $D = (-2)^2 - 4 \cdot p \cdot p = 4 - 4p^2$

1. $D = 0$

$4 - 4p^2 = 0$

$4(1 - p^2) = 0$

$4 \cdot (1-p)(1+p) = 0$

$p = \pm 1$

ak $p = 1$

$x = \frac{2}{2p} = \frac{1}{=}$

ak $p = -1$

$x = \frac{2}{2p} = \frac{-1}{=}$

2. $D < 0$

$4 - 4p^2 < 0$

$p \in (-\infty; -1) \cup (1, \infty)$

$\Downarrow$

$x \in \emptyset$

3. $D > 0$

$4 - 4p^2 > 0$

$p \in (-1; 1)$

$\Downarrow$

$x_{1,2} = \frac{2 \pm 2\sqrt{1-p^2}}{2p} = \frac{1 \pm \sqrt{1-p^2}}{p}$

p	K
0	$\{0\}$
1	$\{1\}$
-1	$\{-1\}$
$(-\infty; -1) \cup (1; \infty)$	$\emptyset$
$(-1; 1)$	$\frac{1 \pm \sqrt{1-p^2}}{p}$

KVADRATICKÁ S PARAMETROM

$(1+a)x^2 - 3ax - 4a = 0$

↳ je kvadratická?

- ak $1+a = 0$, t.j. $a = -1 \Rightarrow \quad 0 - 3.(-1).x - 4.(-1) = 0$

$3x + 4 = 0$

$x = -\frac{4}{3}$

- ak $a \neq -1 \Rightarrow$ kvadratická

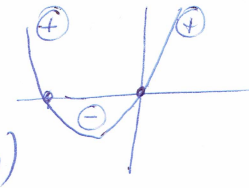
$a_D = 1+a \qquad b_D = -3a \qquad c_D = -4a$

$D = (-3b)^2 - 4.(1+a).(-4a) = 9b^2 + 16a + 16a^2 = 25a^2 + 16a = a.(25a+16)$

$D = 0 \Leftrightarrow 25a^2 + 16a = 0 \Leftrightarrow a.(25a+16) = 0 \Leftrightarrow a = 0$ alebo $a = -\frac{16}{25}$

$D < 0 \Leftrightarrow a \in (-\frac{16}{25}; 0)$

$D < 0 \Leftrightarrow a \in (-\infty; -\frac{16}{25}) \cup (0; \infty)$



$D = 0$

$a = 0; a = -\frac{16}{25}$

$D < 0$

$a \in (-\frac{16}{25}; 0)$

$D > 0$

$a \in (-\infty; -\frac{16}{25}) \cup (0; \infty)$ okrem -1 !

$x = -\frac{b_D}{a_D}$

$x = +\frac{+3a}{1+a}$

$x \in \emptyset$

$x_{1,2} = \frac{-b_D \pm \sqrt{D}}{2a_D}$

$x_{1,2} = \frac{+3a \pm \sqrt{25a^2+16}}{2.(1+a)}$

a	x
-1	$-\frac{4}{3}$
0	$\frac{3.0}{1+0} = 0$
$-\frac{16}{25}$	$\frac{3.(-\frac{16}{25})}{1-\frac{16}{25}} = -\frac{48}{25}.\frac{25}{25-16} = -\frac{48}{9} = -\frac{16}{3}$
$(-\frac{16}{25}; 0)$	$\emptyset$
$(-\infty; -1) \cup (-1; -\frac{16}{25}) \cup (0; \infty)$	$\frac{3a \pm \sqrt{25a^2+16}}{2+2a}$

a_D, b_D, c_D – koef. pre diskriminant

Kvadratická s parametrom

$sx^2 + 1 = 0$ $\quad a = s \quad b = 0 \quad c = 1$

! ak pri x^2 je koeficient 0 $\Rightarrow$ nie je to kvadratická

$0 \cdot x^2 + 1 = 0 \Rightarrow 1 = 0 \Rightarrow$ n. r ①

ak $s = 0$

$s \neq 0$

$D = 0^2 - 4 \cdot s \cdot 1 = -4s$

$D = 0$ ~~$-4s < 0$~~

$-4s = 0$

$s = 0$

$-4s < 0$

$s > 0$

$D > 0$

$-4s > 0$

$s < 0$

②

$x = -\frac{b}{2a} = -\frac{0}{2s}$

$x \in \emptyset$

$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$

$x_{1,2} = \frac{-0 \pm \sqrt{-4s}}{2 \cdot s}$

$x_{1,2} = \frac{\pm 2\sqrt{-s}}{2s} = \frac{\pm\sqrt{-s}}{s}$

neplatí, lebo $s = 0$ vypadlo na začiatku preto je to lineárna

Pozor!

z ① vyplýva, že ② neplatí!

s	x
$(-\infty; 0)$	$\frac{\pm\sqrt{-s}}{s}$
0	$\emptyset$
$(0; \infty)$	$\emptyset$

23/186

Kvadratická s parametrom

$tx^2 + 2tx + 4 = 0$ $\quad a = t \quad b = 2t \quad c = 4$

! nenulový koeficient pri x^2

ak $t = 0$: $\quad 0x^2 + 2.0x + 4 = 0 \Rightarrow$ n. r.

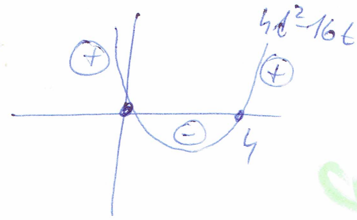
ak $t \neq 0$:

$D = (2t)^2 - 4.t.4 = 4t^2 - 16t = 4t.(t-4)$

$D = 0 \Leftrightarrow 4t.(t-4) = 0 \Leftrightarrow t = 0$ alebo $t = 4$

$D < 0 \Leftrightarrow t \in (0;4)$

$D > 0 \Leftrightarrow t \in (-\infty;0) \cup (4;\infty)$



23/11c)

$D = 0$: $t = 0$ alebo $t = 4$

$x = \frac{-b}{2a} = \frac{-2t}{2.t} = -1$

$D < 0$: $t \in (0;4)$ $\Rightarrow \emptyset$

$D > 0$: $t \in (-\infty;0) \cup (4;\infty)$

$x_{1,2} = \frac{-2t \pm \sqrt{4t^2 - 16t}}{2t}$

$x_{1,2} = \frac{-2t \pm 2\sqrt{t^2 - 4t}}{2t}$

$x_{1,2} = \frac{-t \pm \sqrt{t^2 - 4t}}{t}$

! $t = 0$ nesedí s tým, že je kvadrat. !

preto pre $t = 0 \Rightarrow \emptyset$

pre $t = 4 \Rightarrow x = -1$

t	x
~~$(-\infty;0)\cup(4;\infty)$~~ $(-\infty;0)\cup(4;\infty)$	$\frac{-t \pm \sqrt{t^2-4t}}{t}$
$(0;4)$	$\emptyset$
0	$\emptyset$
4	-1

tieto dva riadky možno napísať

t	x
$\langle 0;4)$	$\emptyset$